

The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2 *Solution 1.0*

Date: November 16, 2021

Course: EE 313 Evans

Name: _____ **Tolkien,** **J. R. R.**
Last, First

- This in-person exam is scheduled to last 75 minutes.
- Open books, open notes, and open class materials, including homework assignments and solution sets and previous midterm exams and solutions.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- ***Please disable all wireless connections on your calculator(s) and computer system(s).***
- Please mute all computer systems.
- Please turn off all phones.
- No headphones are allowed.
- All work should be performed on the midterm exam. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

	<i>Problem</i>	<i>Point Value</i>	<i>Your score</i>	<i>Topic</i>
Arwen	1	27		System Properties
Gandalf	2	27		Convolution
Frodo	3	26		Highpass Filters
My Precious	4	20		Wireless Localization
	<i>Total</i>	100		

Problem 2.1. System Properties. 27 points.

Each discrete-time system has input $x[n]$ and output $y[n]$, and $x[n]$ and $y[n]$ might be complex-valued. Each continuous-time system has input $x(t)$ and output $y(t)$, and $x(t)$ and $y(t)$ might be complex-valued. Determine if each system is linear or nonlinear, time-invariant or time-varying, and stable or unstable.

You must either prove that the system property holds in the case of linearity, time-invariance, or stability, or provide a counter-example that the property does not hold. Providing an answer without any justification will earn 0 points.

Part	System Name	System Formula	Linear?	Time-Invariant?	Stable?
(a)	Averaging Filter	$y[n] = x[n] + x[n-1]$ for $n \geq 0$ and $x[-1] = 0$	Yes	Yes	Yes
(b)	Frequency modulation	$y(t) = \cos\left(2\pi f_c t + 2\pi k_f \int_0^t x(\lambda) d\lambda\right)$ for $t \geq 0$. f_c and k_f are real-valued constants.	No	No	Yes
(c)	Time Reversal	$y(t) = x(-t)$ for $-\infty < t < \infty$	Yes	No	Yes

Linearity. We'll first apply the all-zero input test. If the output is not zero for all time, then the system is not linear. Otherwise, we'll have to apply the definitions for homogeneity and additivity. All-zero input test is a special case of homogeneity $a x(t) \rightarrow a y(t)$ when the constant $a = 0$.

Stability. Bounded input $|x(t)| \leq B < \infty$ would give bounded output $|y(t)| \leq C < \infty$.

(a) Averaging filter: $y[n] = x[n] + x[n-1]$ for $n \geq 0$ and $x[-1] = 0$. 9 points.

Homework 5.2(b)

Linearity: Passes all-zero input test.

- **Homogeneity:** Input $a x[n]$. Output $y_{scaled}[n] = (a x[n]) + (a x[n])_{n \rightarrow n-1} = a x[n] + a x[n-1] = a y[n]$. YES.

- **Additivity.** Input $x_1[n] + x_2[n]$. Output is

$$y_{additive}[n] = (x_1[n] + x_2[n]) + (x_1[n] + x_2[n])_{n \rightarrow n-1} = (x_1[n] + x_2[n]) + (x_1[n-1] + x_2[n-1]) = x_1[n] + x_1[n-1] + x_2[n] + x_2[n-1] = y_1[n] + y_2[n]. \text{ YES.}$$

T-I: Input $x[n - n_0]$. Output $y_{shifted}[n] = x[n - n_0] + x[n - n_0 - 1] = y[n - n_0]$

Stability: $|y[n]| = |x[n] + x[n-1]| \leq |x[n]| + |x[n-1]| = B + B$. YES.

(b) Frequency Modulation: $y(t) = \cos\left(2\pi f_c t + 2\pi k_f \int_0^t x(\lambda) d\lambda\right)$ for $t \geq 0$. 9 points.

Homework 7.3(b)

Linearity: When $x(t) = 0$, $y(t) = \cos(2\pi f_c t)$. Does not pass all-zero input test. NOT LINEAR.

T-I: Input $x(t - t_0)$. Output $y_{shifted}(t) = \cos\left(2\pi f_c t + 2\pi k_f \int_0^{t-t_0} x(\lambda) d\lambda\right)$. Time t in the $2\pi f_c t$ term in the argument of the cosine function does not shift. NO.

Stability: The output of the cosine is always in the range $[-1, 1]$ which is bounded. YES.

(c) Time reversal: $y(t) = x(-t)$ for $-\infty < t < \infty$. 9 points.

SP First, p. 116

Homework 7.3(d)

Linearity: Passes all-zero input test.

- **Homogeneity:** $y_{scaled}(t) = (a x(t))_{t \rightarrow -t} = a x(-t) = a y(t)$. YES.

- **Additivity:** $y_{additive}(t) = (x_1(t) + x_2(t))_{t \rightarrow -t} = x_1(-t) + x_2(-t) = y_1(t) + y_2(t)$. YES.

T-I: Input $x(t - t_0)$. Output $y_{shifted}(t) = (x(t - t_0))_{t \rightarrow -t} = x(-t - t_0) \neq y(t - t_0) = x(-t + t_0)$

Stability: $|y(t)| = |x(-t)| \leq B$. YES.

Problem 2.2 Convolution. 27 points.

In continuous-time systems, a linear time-invariant (LTI) integrator is a building block. Its input-output relationship is

$$y(t) = \int_{-\infty}^t x(\tau) d\tau$$

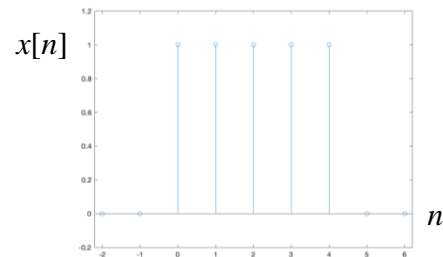
The impulse response is $h(t) = u(t)$ where $u(t)$ is 1 for $t \geq 0$ and 0 otherwise.

The equivalent system in discrete time is the LTI running summation system. Its input-output relationship is

$$y[n] = \sum_{m=-\infty}^n x[m]$$

The impulse response is $h[n] = u[n]$ where $u[n]$ is 1 for $n \geq 0$ and 0 otherwise.

- (a) Plot the output $y[n]$ of the LTI running summation when the input is a five-point rectangular pulse $x[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4]$? On the right, $x[n]$ is plotted for $-2 \leq n \leq 6$. 9 points.



From the convolution definition,

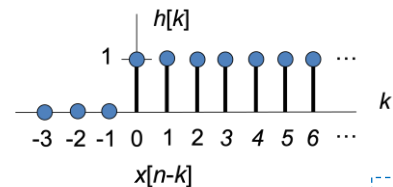
$$y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k] = \sum_{k=-\infty}^{\infty} u[k] x[n-k]$$

We hold $h[k]$ in place, and flip-and-slide $x[n-k]$ across $h[k]$:

Case I: No Overlap. $y[n] = 0$ for $n < 0$.

Case II: Partial Overlap. $y[n] = (n+1)$ for $0 \leq n \leq 3$.

Case III: Complete Overlap. $y[n] = 5$ for $n > 3$.



HW 6.1

Tune-Up #6

F17 Midterm 2.1b

F17 Midterm 2.2b

- (b) Give a formula for the output $y[n]$ of the LTI running summation when the input is $x[n] = u[n]$. The formula should be valid for $-\infty < n < \infty$. 9 points.

From the convolution definition,

$$y[n] = h[n] * x[n] = u[n] * u[n] = \sum_{k=-\infty}^{\infty} u[k] u[n-k] = \sum_{k=0}^n 1 = (n+1) u[n]$$

Simplification of the limits of summation is because $u[k]$ is 1 for $k \geq 0$ and 0 otherwise, whereas $u[n-k]$ is 1 for $n-k \geq 0$ or $k \leq n$. Note $n \geq 0$ due to $k \geq 0$; hence, the $u[n]$ term.

- (c) Give a formula for the output $y(t)$ of the LTI integrator when the input is $x(t) = u(t)$. The formula should be valid for $-\infty < t < \infty$. 9 points.

From the convolution definition,

$$y(t) = h(t) * x(t) = u(t) * u(t) = \int_{-\infty}^{\infty} u(\lambda) u(t-\lambda) d\lambda = \int_0^t d\lambda = t u(t)$$

Simplification in the limits of integration is because $u(\lambda)$ is 1 for $\lambda \geq 0$ and 0 otherwise, whereas $u(t-\lambda)$ is 1 for $t-\lambda \geq 0$ or $\lambda \leq t$. Note $t \geq 0$ due to $\lambda \geq 0$; hence, the $u(t)$ term.

HW 7.4(b)iii

Problem 2.3. Highpass Filters. 26 points.

A highpass filter passes high frequencies and attenuates low frequencies.

Shown below are the pole-zero plots for two discrete-time linear time-invariant (LTI) highpass filters.

(a) LTI first-order finite impulse response (FIR) filter.

Pole-zero plot on the right. Zero at $z = 1$ and pole at $z = 0$.

- i. Write the transfer function $H(z)$ including the region of convergence. 6 points.

$$H(z) = \frac{z - 1}{z - 0} = \frac{z - 1}{z} = 1 - z^{-1} \text{ for } z \neq 0$$

- ii. Give a formula for the frequency response. 3 points.

Since the region of convergence includes $z = e^{j\hat{\omega}}$, we substitute $z = e^{j\hat{\omega}}$ to convert to a frequency response:

$$H_{freq}(\hat{\omega}) = H(z)_{z=e^{j\hat{\omega}}} = \frac{e^{j\hat{\omega}} - 1}{e^{j\hat{\omega}}} = 1 - e^{-j\hat{\omega}}$$

- iii. Plot the magnitude response. 4 points.

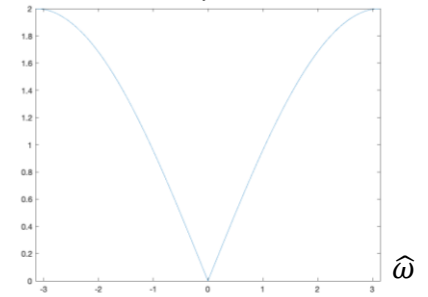
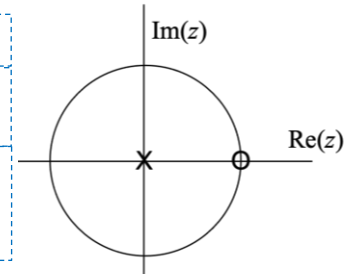
$$|H_{freq}(\hat{\omega})| = \left| \frac{e^{j\hat{\omega}} - 1}{e^{j\hat{\omega}}} \right| = \frac{|e^{j\hat{\omega}} - 1|}{|e^{j\hat{\omega}}|} = |e^{j\hat{\omega}} - 1|$$

$$\omega = 0: |H_{freq}(\hat{\omega})| = |e^{j0} - 1| = 0$$

$$\omega = \frac{\pi}{2}: |H_{freq}(\hat{\omega})| = \left| e^{j\frac{\pi}{2}} - 1 \right| = \sqrt{2}$$

$$\omega = \pi: |H_{freq}(\hat{\omega})| = |e^{j\pi} - 1| = 2$$

Homework 6.3(b)

Homework 7.2
Pole-Zero Plot #1First-Order
Difference Filter
 $y[n]=x[n]-x[n-1]$ 

```
w = -pi : 0.001 : pi;
z0 = 1;
p0 = 0;
H = ( exp(j*w) - z0 ) ./
    (exp(j*w) - p0);
plot(w, abs(H));
xlim( [-pi pi] );
```

(b) LTI first-order infinite impulse response (IIR) filter.

Pole-zero plot on the right. Pole at $z = -0.9$ and zero at $z = 0$.

- i. Write the transfer function $H(z)$ including the region of convergence. 6 points.

$$H(z) = \frac{z - 0}{z - (-0.9)} = \frac{z}{z + 0.9} = \frac{1}{1 + 0.9z^{-1}} \text{ for } |z| > 0.9$$

- ii. Give a formula for the frequency response. 3 points.

Since the region of convergence includes $z = e^{j\hat{\omega}}$, we can substitute $z = e^{j\hat{\omega}}$ to convert to a frequency response:

$$H_{freq}(\hat{\omega}) = H(z)_{z=e^{j\hat{\omega}}} = \frac{e^{j\hat{\omega}}}{e^{j\hat{\omega}} + 0.9} = \frac{e^{j\hat{\omega}}}{1 + 0.9e^{-j\hat{\omega}}}$$

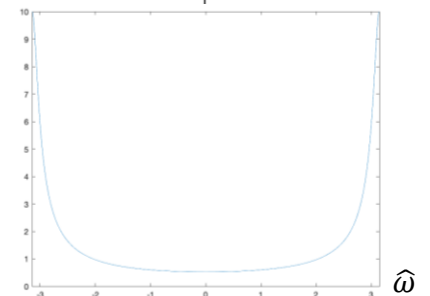
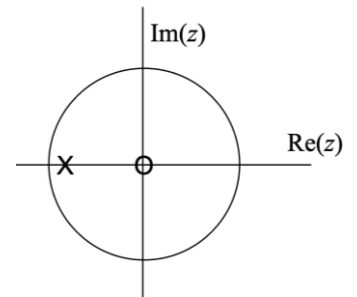
- iii. Plot the magnitude response. 4 points.

$$|H_{freq}(\hat{\omega})| = \left| \frac{e^{j\hat{\omega}}}{e^{j\hat{\omega}} - (-0.9)} \right| = \frac{1}{|e^{j\hat{\omega}} - (-0.9)|}$$

$$\omega = 0: |H_{freq}(\hat{\omega})| = \frac{1}{|e^{j0} - (-0.9)|} \approx 0.526$$

$$\omega = \frac{\pi}{2}: |H_{freq}(\hat{\omega})| = \frac{1}{\left| e^{j\frac{\pi}{2}} - (-0.9) \right|} \approx 0.743$$

$$\omega = \pi: |H_{freq}(\hat{\omega})| = \frac{1}{|e^{j\pi} - (-0.9)|} = 10$$

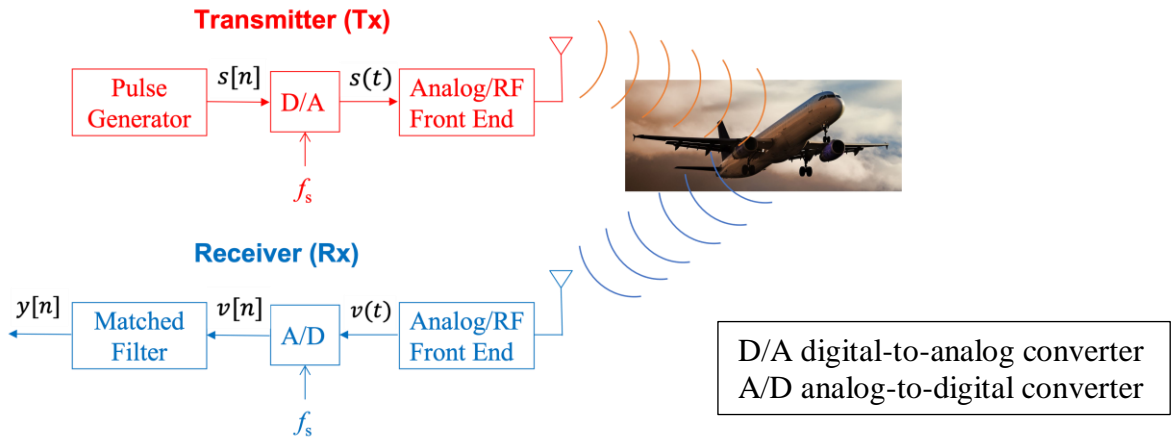
Homework 7.2
Pole-Zero Plot #1

```
w = -pi : 0.001 : pi;
z0 = 0;
p0 = -0.9;
H = ( exp(j*w) - z0 ) ./
    (exp(j*w) - p0);
plot(w, abs(H));
xlim( [-pi pi] );
```

Problem 2.4 Wireless Localization. 20 points.

Mini-Project #2 used radar (electromagnetic waves and signal processing) to locate an object in an environment. Applications include automotive and 6G cellular communication systems.

A block diagram for the radar system to determine the distance to the object follows.



The radar system estimates the round-trip time T_d from the transmitter to the object and from the object to the receiver. Transmitter and receiver are in the same location. Distance d can be computed using $d = \frac{1}{2} c T_d$ where c is the speed of propagation of electromagnetic waves in air (speed of light).

(a) What are the advantages of using a chirp pulse for the transmitted waveform? 6 points.

“A chirp pulse linearly sweeps a range of frequencies over an interval of time. A chirp is resilient to many impairments experienced by propagating signals including thermal noise, delays, frequency distortion, and Doppler shifts. [7] Chirp signals can be used not only to localize an object, but also estimate its velocity. [8]” From the [Mini-Project #2 Solution](#).

(b) Describe the role of the matched filter in the receiver. Why is it called a matched filter? What is being matched? 6 points.

Matched filtering detects a known pulse by correlating the received signal with the known pulse and by processing the points in the absolute value of the output that exceed a threshold.

A matched filter gets its name from its impulse response being derived from the pulse it is correlating against. To find the impulse response, the pulse shape is flipped in time, delayed, conjugated, and scaled by a constant. From the [Mini-Project #2 Solution](#).

(c) How did the radar system use the chirp pulse and matched filter to estimate the round-trip time T_d from the transmitter to the object and from the object to the receiver? 8 points.

The radar receiver processed the output of the matched filter to find “the time at which the first peak occurs and subtract the worst-case delay through the matched filter from the time” to estimate the round-trip time T_d . From the [Mini-Project #2 Solution](#).